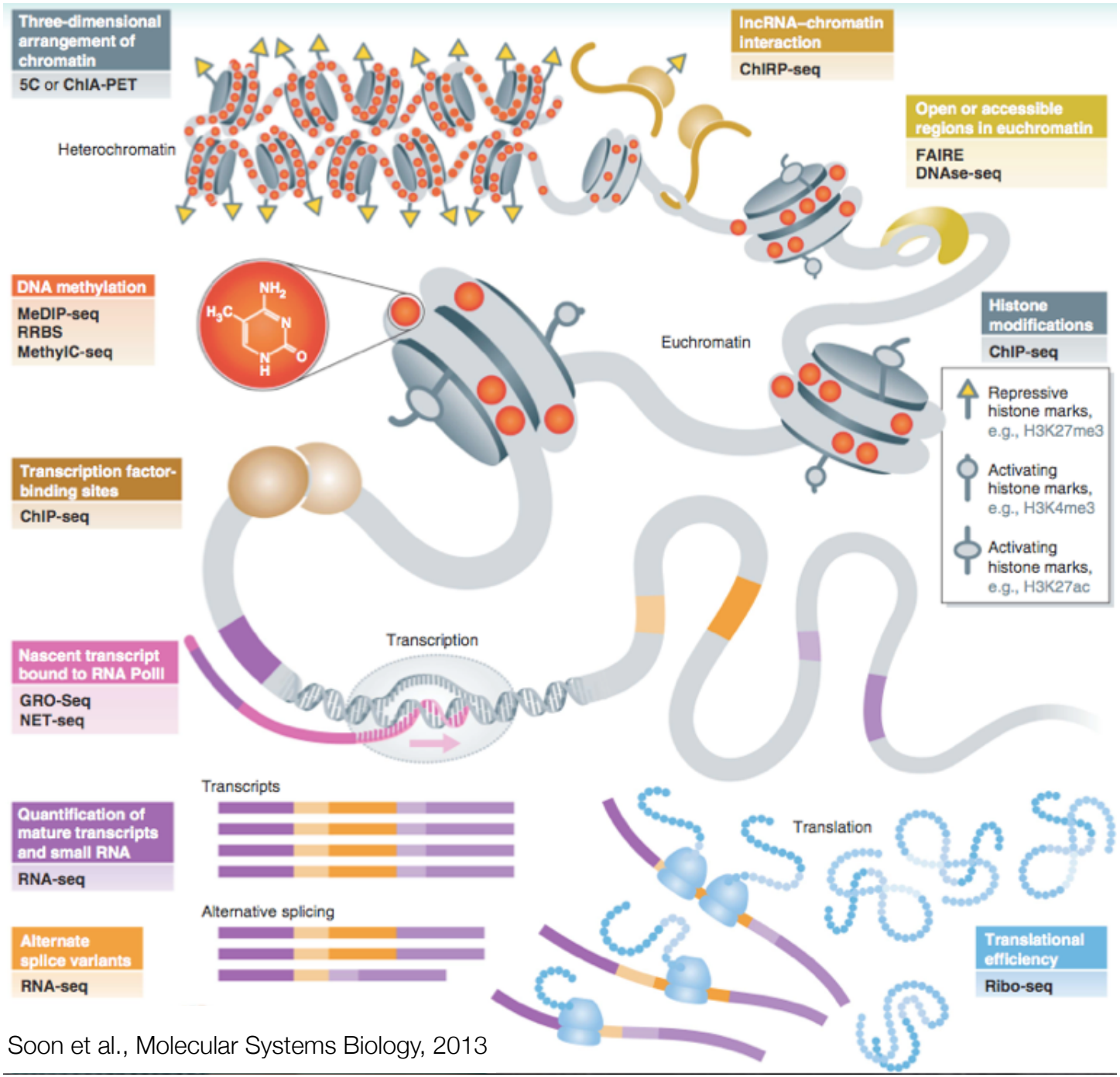
Sorting, Searching, & Aligning

Michael Schatz

Bioinformatics Lecture 1
Quantitative Biology 2013



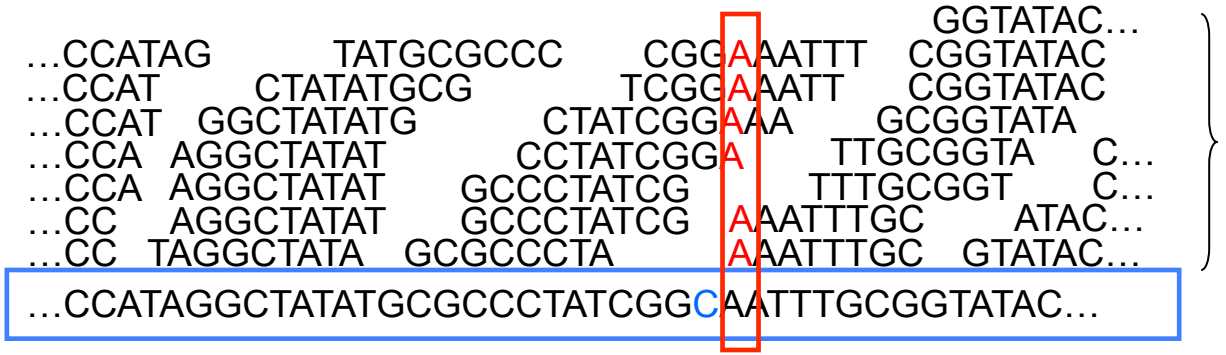
Soon et al., Molecular Systems Biology, 2013

Sequencing Assays

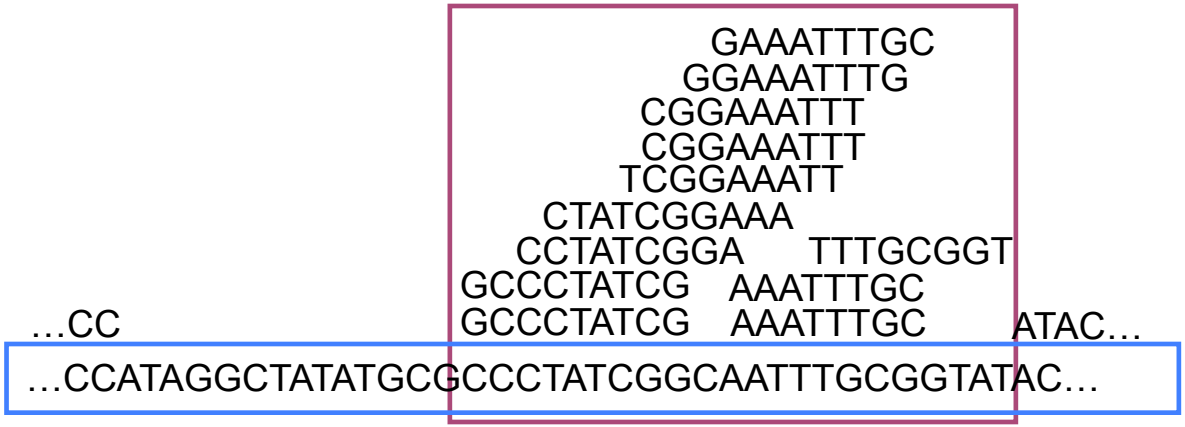
1. Gregory E. Crawford et al., "Genome-wide Mapping of DNase Hypersensitive Sites Using Massively Parallel Signature Sequencing (MPSS)," Genome Research 16, no. 1 (January 1, 2006): 123–131, doi:10.1101/gr.4074106.
2. David S. Johnson et al., "Genome-Wide Mapping of in Vivo Protein-DNA Interactions," Science 316, no. 5830 (June 8, 2007): 1497–1502, doi:10.1126/science.1141319.
3. Tarjei S. Mikkelsen et al., "Genome-wide Maps of Chromatin State in Pluripotent and Lineage-committed Cells," Nature 448, no. 7153 (August 2, 2007): 553–560, doi:10.1038/nature06008.
4. Nathan A. Baird et al., "Rapid SNP Discovery and Genetic Mapping Using Sequenced RAD Markers," PLoS ONE 3, no. 10 (October 13, 2008): e3376, doi:10.1371/journal.pone.0003376.
5. Leighton J. Core, Joshua J. Waterfall, and John T. Lis, "Nascent RNA Sequencing Reveals Widespread Pausing and Divergent Initiation at Human Promoters," Science 322, no. 5909 (December 19, 2008): 1845–1848, doi:10.1126/science.1162228.
6. Thomas A. Down et al., "A Bayesian Deconvolution Strategy for Immunoprecipitation-based DNA Methylome Analysis," Nature Biotechnology 26, no. 7 (July 2008): 779–785, doi:10.1038/nbt1414.
7. Ali Mortazavi et al., "Mapping and Quantifying Mammalian Transcriptomes by RNA-Seq," Nature Methods 5, no. 7 (July 2008): 621–628, doi:10.1038/nmeth.1226.
8. Alayne L. Brunner et al., "Distinct DNA Methylation Patterns Characterize Differentiated Human Embryonic Stem Cells and Developing Human Fetal Liver," Genome Research 19, no. 6 (June 1, 2009): 1044–1056, doi:10.1101/gr.088773.108.
9. Melissa J. Fullwood et al., "An Oestrogen-receptor-α-bound Human Chromatin Interactome," Nature 462, no. 7269 (November 5, 2009): 58–64, doi:10.1038/nature08497.
10. Jay R. Hesselberth et al., "Global Mapping of protein-DNA Interactions in Vivo by Digital Genomic Footprinting," Nature Methods 6, no. 4 (April 2009): 283–289, doi:10.1038/nmeth.1313.
11. Nicholas T. Ingolia et al., "Genome-Wide Analysis in Vivo of Translation with Nucleotide Resolution Using Ribosome Profiling," Science 324, no. 5924 (April 10, 2009): 218–223, doi:10.1126/science.1168978.
12. Gemma C. Langridge et al., "Simultaneous Assay of Every Salmonella Typhi Gene Using One Million Transposon Mutants," Genome Research (October 13, 2009), doi:10.1101/gr.097097.109.
13. Erez Lieberman-Aiden et al., "Comprehensive Mapping of Long-Range Interactions Reveals Folding Principles of the Human Genome," Science 326, no. 5950 (October 9, 2009): 289–293, doi:10.1126/science.1181369.
14. Ryan Lister et al., "Human DNA Methylomes at Base Resolution Show Widespread Epigenomic Differences," Nature 462, no. 7271 (November 19, 2009): 315–322, doi:10.1038/nature08514.
15. Andrew M. Smith et al., "Quantitative Phenotyping via Deep Barcode Sequencing," Genome Research (July 21, 2009), doi:10.1101/

Short Read Applications

- Genotyping: Identify Variations



- *-seq: Classify & measure significant peaks



Exact Matching Review & Overview

Where is GATTACA in the human genome?

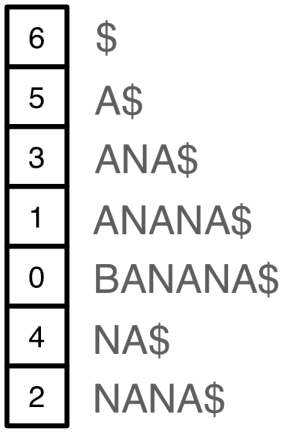
Brute Force (3 GB)

```
BANANA
BAN
  ANA
    NAN
      ANA
```

Naive

Slow & Easy

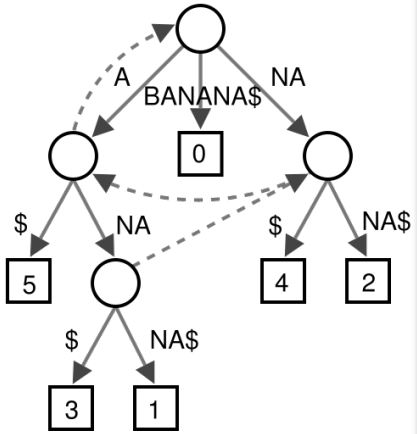
Suffix Array (>15 GB)



Vmatch, PacBio Aligner

Binary Search

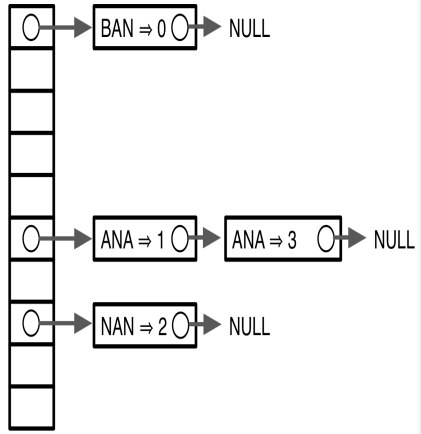
Suffix Tree (>51 GB)



MUMmer, MUMmerGPU

Tree Searching

Hash Table (>15 GB)



BLAST, MAQ, ZOOM, RMAP, CloudBurst

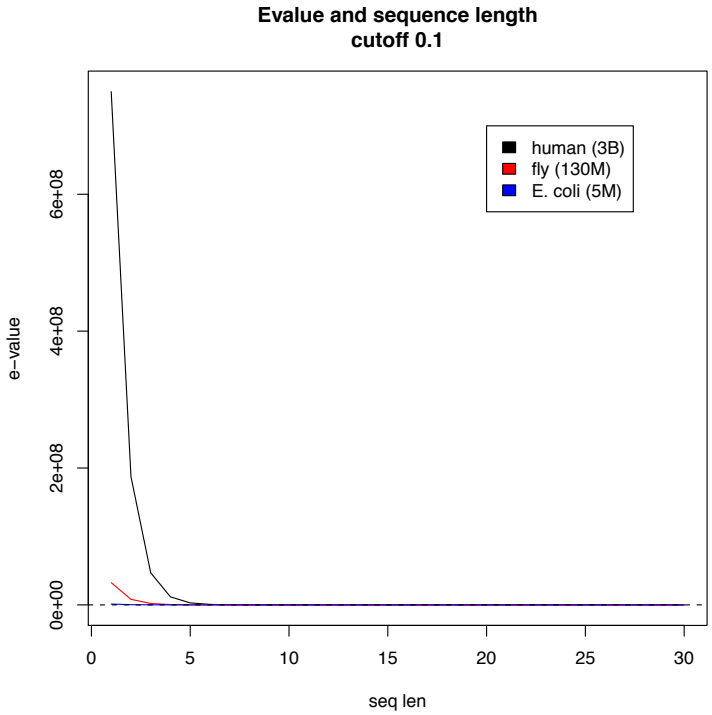
Seed-and-extend

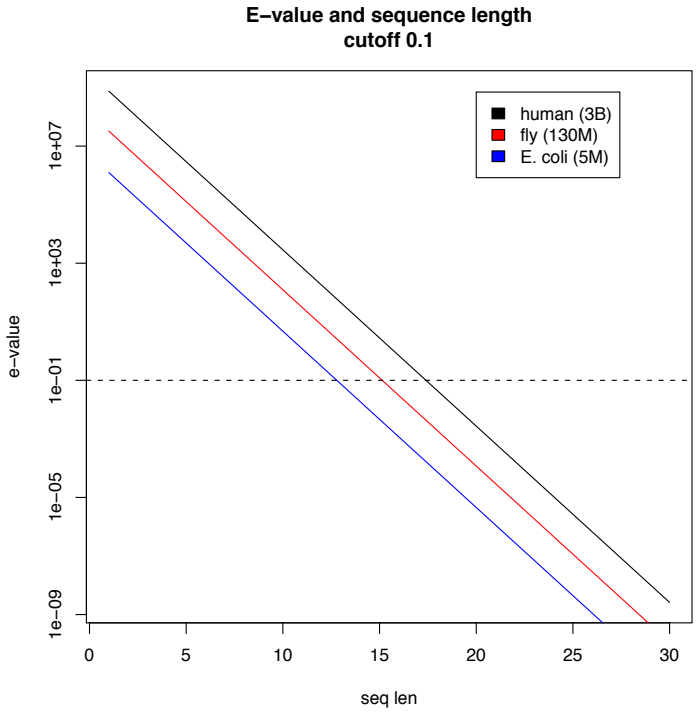
*** These are general techniques applicable to any search problem ***

Expected Occurrences

The expected number of occurrences (e-value) of a given sequence in a genome depends on the length of the genome and inversely on the length of the sequence

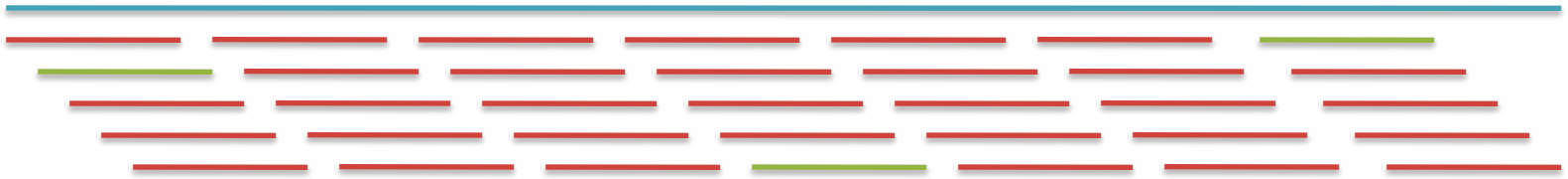
- 1 in 4 bases are G, 1 in 16 positions are GA, 1 in 64 positions are GAT
- 1 in 16,384 should be GATTACA
- $E=(n-m+1)/(4^m)$ [183,105 expected occurrences]





[Challenge Question: What is the expected distribution & variance?]

1. Brute Force



- Brute Force:
  - At every possible offset in the genome:
    - Do all of the characters of the query match?

- Analysis
  - Simple, easy to understand
  - Genome length = n [3B]
  - Query length = m [7]
  - Comparisons: (n-m+1) * m [21B]

- Overall runtime: O(nm)

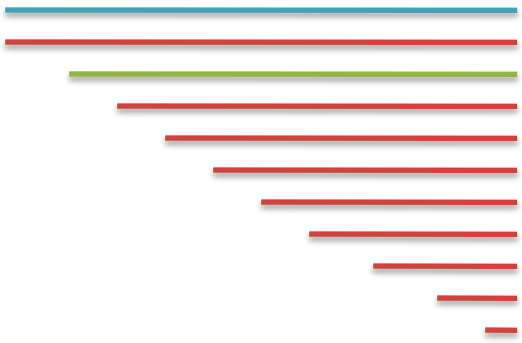
[How long would it take if we double the genome size, read length?]

[How long would it take if we double both?]

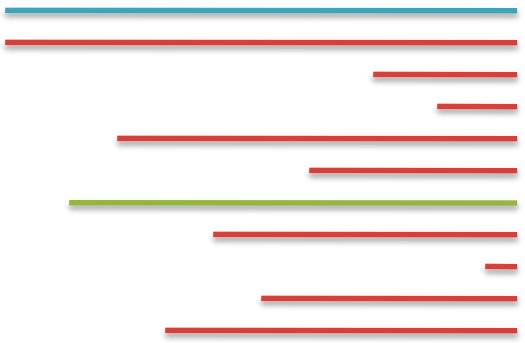
2. Suffix Arrays

- What if we need to check many queries?
  - We don't need to check every page of the phone book to find 'Schatz'
  - Sorting alphabetically lets us immediately skip 96% (25/26) of the book *without any loss in accuracy*

- Sorting the genome: Suffix Array (Manber & Myers, 1991)
  - Sort every suffix of the genome



Split into n suffixes



Sort suffixes alphabetically

Searching the Index

- Strategy 2: Binary search
  - Compare to the middle, refine as higher or lower

- Searching for GATTACA
  - Lo = 1; Hi = 15;

Lo →

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
8	CC…	14
9	GATTACAGATTACC…	2
10	GATTACC…	9
11	TACAGATTACC…	5
12	TACC…	12
13	TGATTACAGATTACC…	1
14	TTACAGATTACC…	4
15	TTACC…	11

Hi →

Searching the Index

- Strategy 2: Binary search
  - Compare to the middle, refine as higher or lower

- Searching for GATTACA
  - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
  - Middle = Suffix[8] = CC

Lo →

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
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10	GATTACC…	9
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  - Compare to the middle, refine as higher or lower

- Searching for GATTACA
  - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
  - Middle = Suffix[8] = CC

    => Higher: Lo = Mid + 1

Lo →

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
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15	TTACC…	11

Hi →

Searching the Index

- Strategy 2: Binary search
  - Compare to the middle, refine as higher or lower

- Searching for GATTACA
  - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
  - Middle = Suffix[8] = CC
    => Higher: Lo = Mid + 1

  - Lo = 9; Hi = 15;

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
8	CC…	14
9	GATTACAGATTACC…	2
10	GATTACC…	9
11	TACAGATTACC…	5
12	TACC…	12
13	TGATTACAGATTACC…	1
14	TTACAGATTACC…	4
15	TTACC…	11

Lo →

Hi →

Searching the Index

- Strategy 2: Binary search
  - Compare to the middle, refine as higher or lower

- Searching for GATTACA
  - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
  - Middle = Suffix[8] = CC

    => Higher: Lo = Mid + 1

  - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
  - Middle = Suffix[12] = TACC

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
8	CC…	14
9	GATTACAGATTACC…	2
10	GATTACC…	9
11	TACAGATTACC…	5
12	TACC…	12
13	TGATTACAGATTACC…	1
14	TTACAGATTACC…	4
15	TTACC…	11

Lo →

Hi →

Searching the Index

- Strategy 2: Binary search
  - Compare to the middle, refine as higher or lower

- Searching for GATTACA
  - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
  - Middle = Suffix[8] = CC
    => Higher: Lo = Mid + 1

  - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
  - Middle = Suffix[12] = TACC
    => Lower: Hi = Mid - 1

  - Lo = 9; Hi = 11;

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
8	CC…	14
9	GATTACAGATTACC…	2
10	GATTACC…	9
11	TACAGATTACC…	5
12	TACC…	12
13	TGATTACAGATTACC…	1
14	TTACAGATTACC…	4
15	TTACC…	11

Lo →

Hi →

Searching the Index

- Strategy 2: Binary search
  - Compare to the middle, refine as higher or lower

- Searching for GATTACA
  - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
  - Middle = Suffix[8] = CC
    => Higher: Lo = Mid + 1

  - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
  - Middle = Suffix[12] = TACC
    => Lower: Hi = Mid - 1

  - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
  - Middle = Suffix[10] = GATTACC

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
8	CC…	14
9	GATTACAGATTACC…	2
10	GATTACC…	9
11	TACAGATTACC…	5
12	TACC…	12
13	TGATTACAGATTACC…	1
14	TTACAGATTACC…	4
15	TTACC…	11

Lo →

Hi →

Searching the Index

- Strategy 2: Binary search
  - Compare to the middle, refine as higher or lower

- Searching for GATTACA
  - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
  - Middle = Suffix[8] = CC
    => Higher: Lo = Mid + 1

  - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
  - Middle = Suffix[12] = TACC
    => Lower: Hi = Mid - 1

  - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
  - Middle = Suffix[10] = GATTACC
    => Lower: Hi = Mid - 1

  - Lo = 9; Hi = 9;

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
8	CC…	14
9	GATTACAGATTACC…	2
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11	TACAGATTACC…	5
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Lo
Hi

Searching the Index

- Strategy 2: Binary search
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- Searching for GATTACA
  - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
  - Middle = Suffix[8] = CC
    => Higher: Lo = Mid + 1

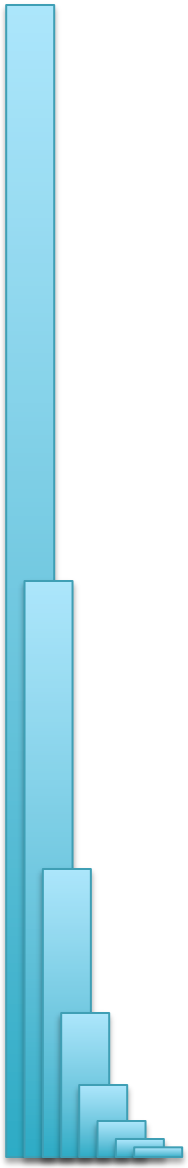
  - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
  - Middle = Suffix[12] = TACC
    => Lower: Hi = Mid - 1

  - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
  - Middle = Suffix[10] = GATTACC
    => Lower: Hi = Mid - 1

  - Lo = 9; Hi = 9; Mid = (9+9)/2 = 9
  - Middle = Suffix[9] = GATTACA…
    => Match at position 2!

#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
8	CC…	14
9	GATTACAGATTACC…	2
10	GATTACC…	9
11	TACAGATTACC…	5
12	TACC…	12
13	TGATTACAGATTACC…	1
14	TTACAGATTACC…	4
15	TTACC…	11

Binary Search Analysis



- Binary Search
  - Initialize search range to entire list
    - mid = (hi+lo)/2; middle = suffix[mid]
    - if query matches middle: done
    - else if query < middle: pick low range
    - else if query > middle: pick hi range
  - Repeat until done or empty range [WHEN?]

- Analysis
  - More complicated method
  - How many times do we repeat?
    - How many times can it cut the range in half?
    - Find smallest x such that: $n/(2^x) \leq 1$; $x = \lg_2(n)$ [32]

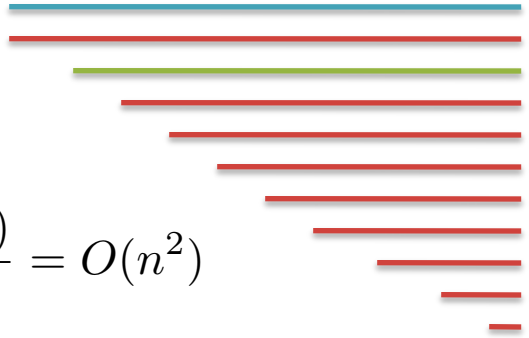
- Total Runtime: O(m lg n)
  - More complicated, but much faster!
  - Looking up a query loops 32 times instead of 3B

[How long does it take to search 6B or 24B nucleotides?]

Suffix Array Construction

- How can we store the suffix array?

  [How many characters are in all suffixes combined?]



$$S = 1 + 2 + 3 + \cdots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = O(n^2)$$

  - Hopeless to explicitly store 4.5 billion billion characters
  - Instead use implicit representation
    - Keep 1 copy of the genome, and a list of sorted offsets
    - Storing 3 billion offsets fits on a server (12GB)

- Searching the array is very fast, but it takes time to construct
  - This time will be amortized over many, many searches
  - Run it once "overnight" and save it away for all future queries

Pos
6
13
8
3
10
15
7
14
2
9
5
12
1
4
11

TGATTACAGATTACC

Sorting

Quickly sort these numbers into ascending order:
14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

[How do you do it?]

6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19
6, 13, 14, 29, 31, 39, 64, 78, 50, 63, 61, 19
6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61
6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61
6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61
6, 13, 14, 19, 29, 31, 39, 50, 64, 78, 63, 61
6, 13, 14, 19, 29, 31, 39, 50, 61, 64, 78, 63
6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78
6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78
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6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78

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Quickly sort these numbers into ascending order:
14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

[How do you do it?]

6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19
6, 13, 14, 29, 31, 39, 64, 78, 50, 63, 61, 19
6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61
6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61
6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61
6, 13, 14, 19, 29, 31, 39, 50, 64, 78, 63, 61
6, 13, 14, 19, 29, 31, 39, 50, 61, 64, 78, 63
6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78
6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78
6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78
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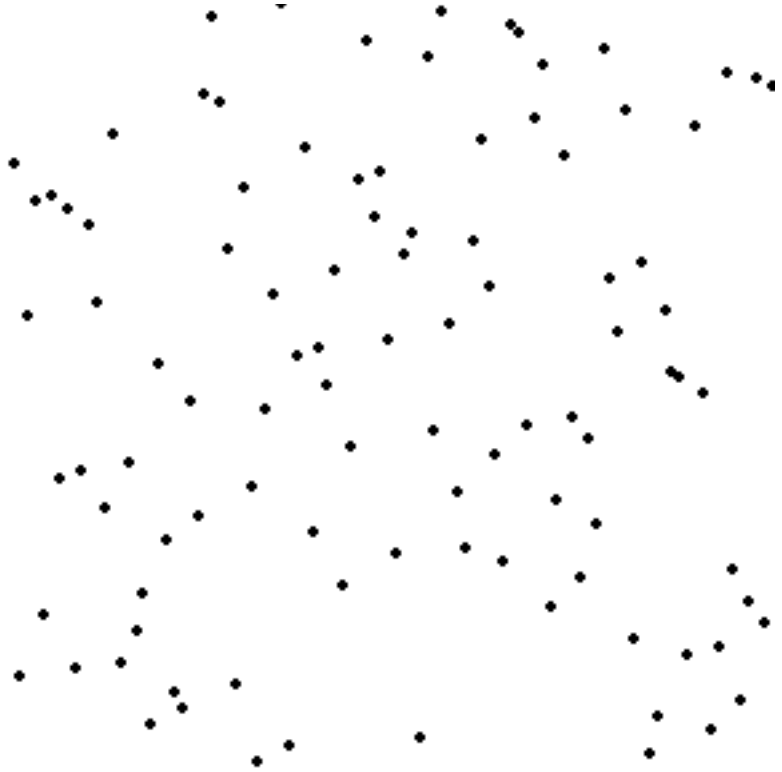


http://en.wikipedia.org/wiki/Selection_sort

Selection Sort Analysis

- Selection Sort (Input: list of n numbers)

```
for pos = 1 to n
    // find the smallest element in [pos, n]
    smallest = pos
    for check = pos+1 to n
        if (list[check] < list[smallest]): smallest = check

    // move the smallest element to the front
    tmp = list[smallest]
    list[pos] = list[smallest]
    list[smallest] = tmp
```

- Analysis

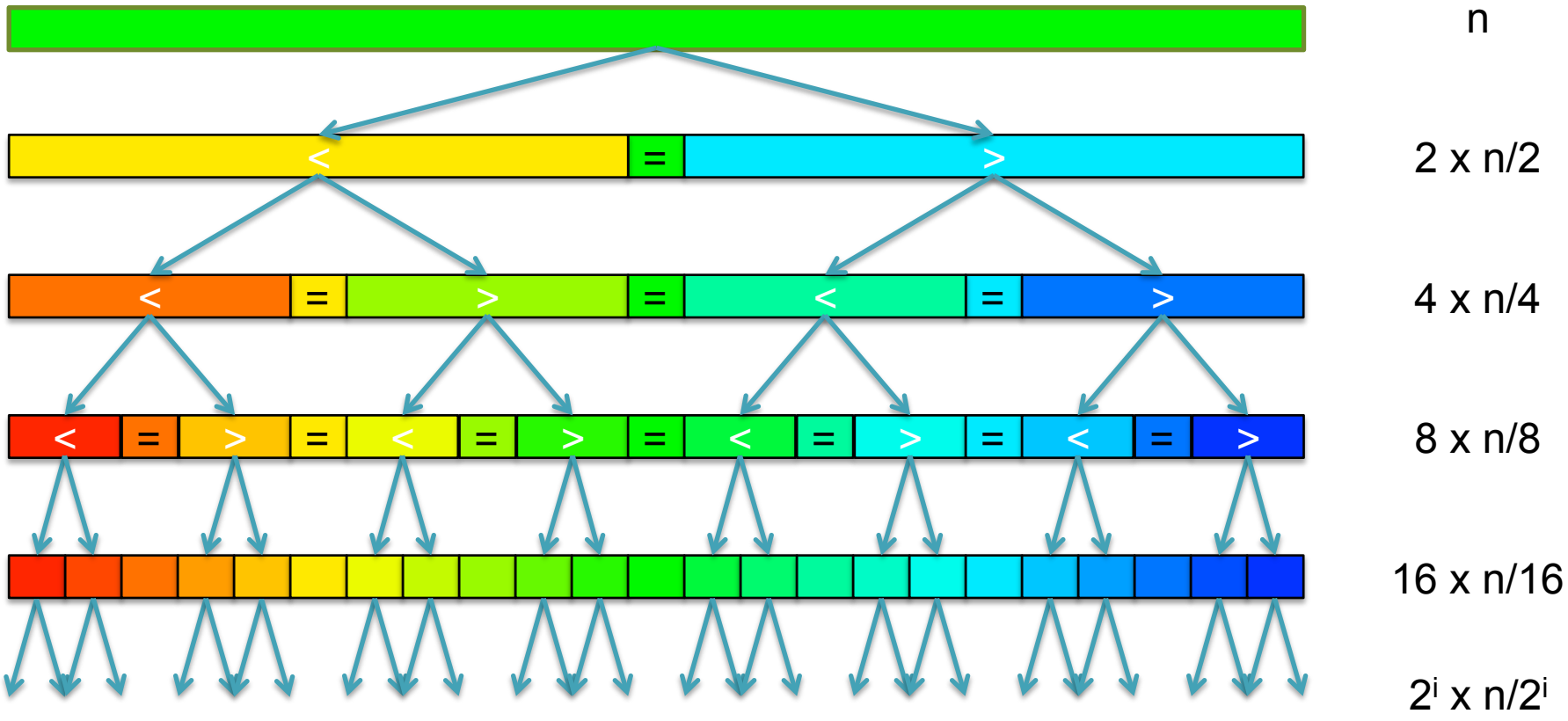
$$T = n + (n-1) + (n-2) + \cdots + 3 + 2 + 1 = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = O(n^2)$$

  - Outer loop: pos = 1 to n
  - Inner loop: check = pos to n
  - Running time: Outer * Inner = $O(n^2)$ [4.5 Billion Billion]

[Challenge Questions: Why is this slow? / Can we sort any faster?]

Divide and Conquer

- Selection sort is slow because it rescans the entire list for each element
  - How can we split up the unsorted list into independent ranges?
  - Hint 1: Binary search splits up the problem into 2 independent ranges (hi/lo)
  - Hint 2: Assume we know the median value of a list



[How many times can we split a list in half?]

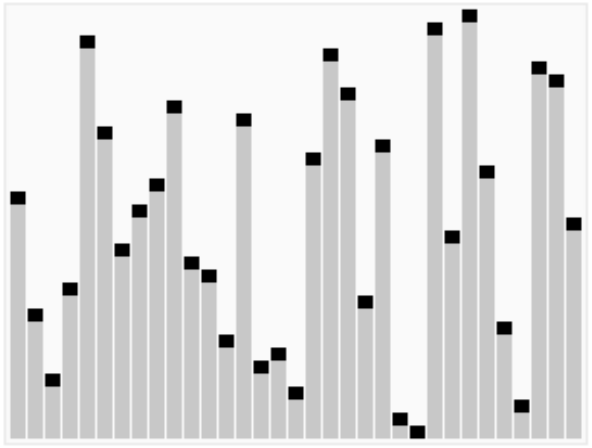
QuickSort Analysis

- QuickSort(Input: list of n numbers)

```
// see if we can quit
if (length(list)) <= 1): return list

// split list into lo & hi
pivot = median(list)
lo = {}; hi = {};
for (i = 1 to length(list))
        if (list[i] < pivot): append(lo, list[i])
        else:                 append(hi, list[i])

// recurse on sublists
return (append(QuickSort(lo), QuickSort(hi))
```



http://en.wikipedia.org/wiki/Quicksort

- Analysis (Assume we can find the median in O(n))

$$T(n) = \begin{cases} O(1) & \text{if } n \leq 1 \\ O(n) + 2T(n/2) & \text{else} \end{cases}$$

$$T(n) = n + 2(\frac{n}{2}) + 4(\frac{n}{4}) + \cdots + n(\frac{n}{n}) = \sum_{i=0}^{lg(n)} \frac{2^i n}{2^i} = \sum_{i=0}^{lg(n)} n = O(n \lg n)$$

[~94B]

QuickSort Analysis

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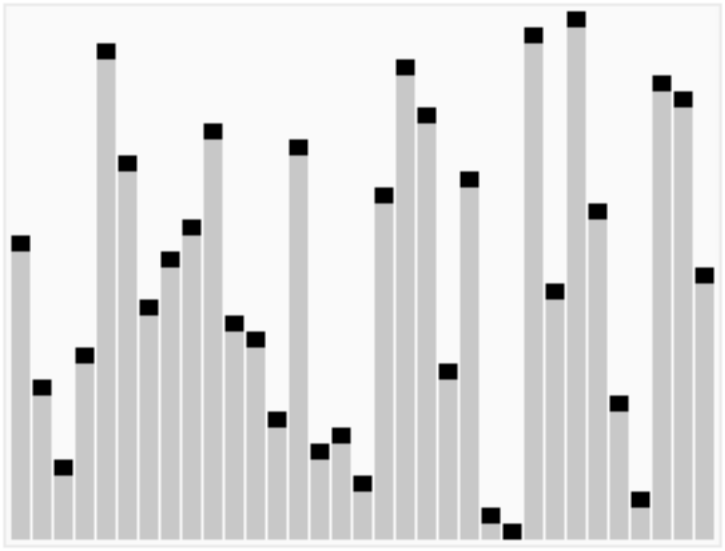
return (append(QuickSort(lo), QuickSort(hi))
```



http://en.wikipedia.org/wiki/Quicksort

- Analysis (Assume we can find the median in O(n))

$$T(n) = \begin{cases} O(1) & \text{if } n \leq 1 \\ O(n) + 2T(n/2) & \text{else} \end{cases}$$

$$T(n) = n + 2(\frac{n}{2}) + 4(\frac{n}{4}) + \cdots + n(\frac{n}{n}) = \sum_{i=0}^{lg(n)} \frac{2^i n}{2^i} = \sum_{i=0}^{lg(n)} n = O(n \lg n)$$

[~94B]

QuickSort in Python

```
list.sort()
```

- The goal of software engineering is to build libraries of correct reusable functions that implement higher level ideas
  - Build complex software out of simple components
  - Software tends to be 90% plumbing, 10% research
  - You still need to know how they work
    - Python requires an explicit representation of the strings

3. Sorting in Linear Time

- Can we sort faster than O(n lg n)?
  - No – Not if we have to compare elements to each other
  - Yes – But we have to 'cheat' and know the structure of the data

Sort these numbers into ascending order:

14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

3. Sorting in Linear Time

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26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

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26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

6,13,14,19,29,31,39,50,61,63,64,78

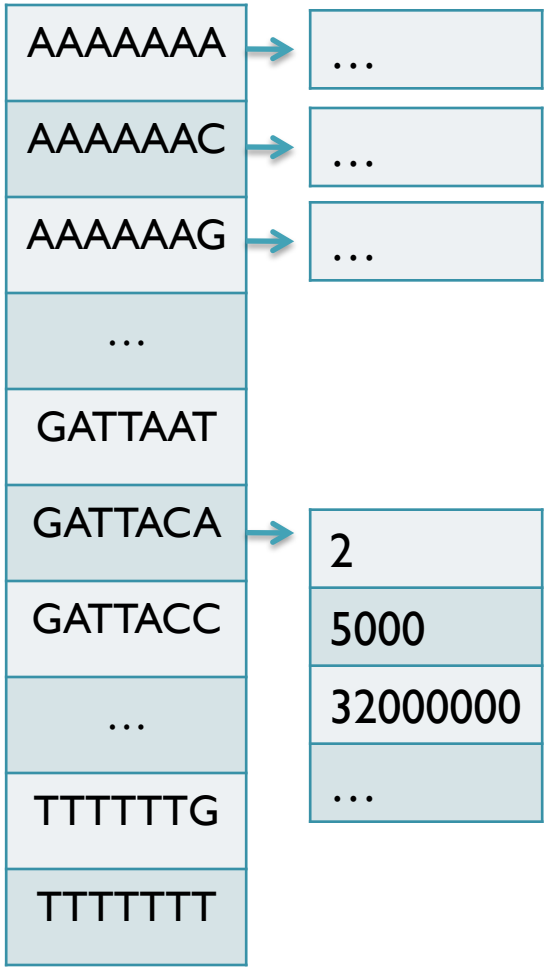
```
for(i = 1 to 100) { cnt[i] = 0; }
for(i = 1 to n) { cnt[list[i]]++; }
for(i = 1 to 100) { while (cnt[i] > 0){print i; cnt[i]--}}
```

[3B instead of 94B]

4. Hashing

- Where is GATTACA in the human genome?
  - Build an inverted index (table) of every kmer in the genome

- How do we access the table?
  - We can only use numbers to index
    - table[GATTACA] <- error, does not compute
  - Encode sequences as numbers
    - Simple: A = 0, C = 1, G = 2, T = 3
      - GATTACA = 2 0 3 3 0 1 0
    - Smart: A = 00_2, C = 01_2, G = 10_2, T = 11_2
      - GATTACA = 10 00 11 11 00 01 00_2 = 9156_{10}
  - Running time
    - Construction: O(n)
    - Lookup: O(1) + O(z)
    - Sorts the genome mers in linear time



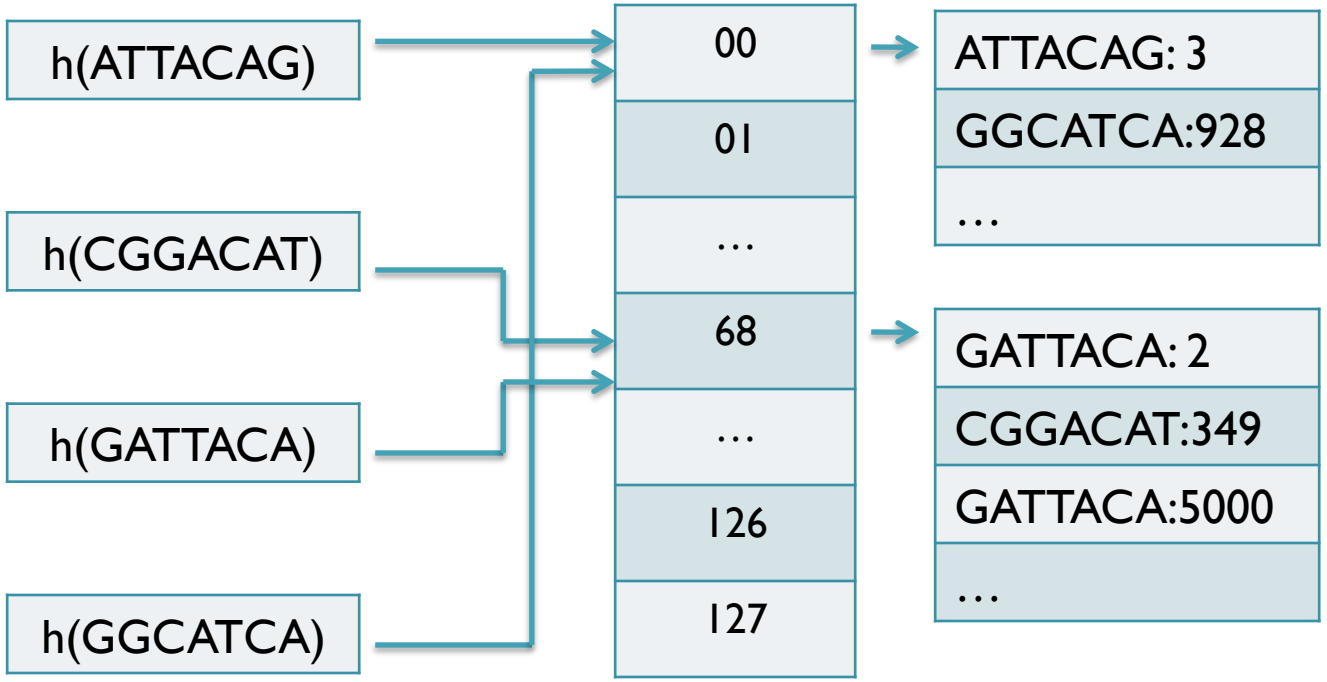
Hash Tables and Hash Functions

- Number of possible sequences of length k = 4^k
  - 4^7 = 16,384 (easy to store)
  - 4^{20} = 1,099,511,627,776 (impossible to directly store in RAM)
    - There are only 3B 20-mers in the genome
      - ⇒ Even if we could build this table, 99.7% will be empty
      - ⇒ But we don't know which cells are empty until we try
- Use a hash function to shrink the possible range
  - Maps a number n in [0,R] to h in [0,H]
    - » Use 128 buckets instead of 16,384, or 1B instead of 1T
  - Division: hash(n) = H * n / R;
    - » hash(GATTACA) = 128 * 9156 / 16384 = 71
  - Modulo: hash(n) = n % H
    - » hash(GATTACA) = 9156 % 128 = 68

[What properties do we want in a hash functions?]

Hash Table Lookup

- By construction, multiple keys have the same hash value
  - Store elements with the same key in a bucket chained together
    - A good hash evenly distributes the values: R/H have the same hash value
  - Looking up a value scans the entire bucket
    - Slows down the search as a function of the hash table load
    - Warning: This complexity is usually hidden in the hash table code



[How many elements do we expect per bucket?]

THE G-NOME PROJECT

Break

Algorithmic challenge

How can we combine the speed of a suffix array or hash table exact match (O(|q|) or O(lg(n)) with the size of a brute force analysis (n bytes)?

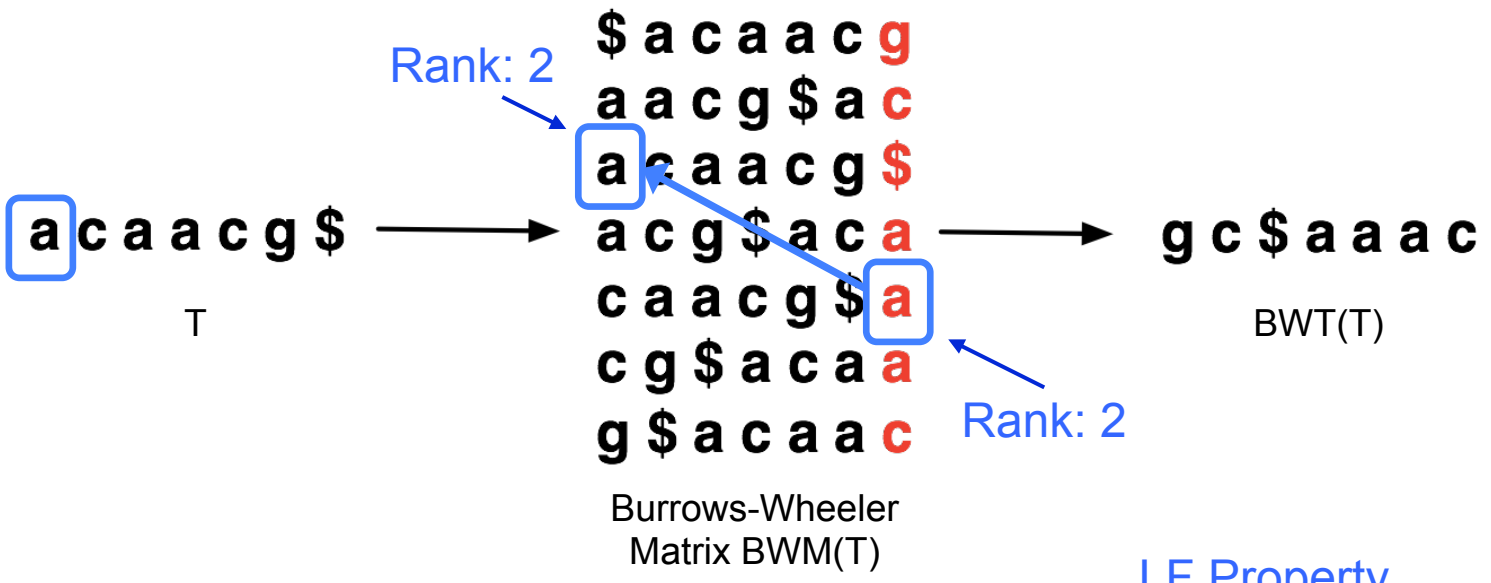
What would such an index look like?

Bowtie: Ultrafast and memory efficient alignment of short DNA sequences to the human genome

Slides Courtesy of Ben Langmead
(langmead@umiacs.umd.edu)

Burrows-Wheeler Transform

- Reversible permutation of the characters in a text



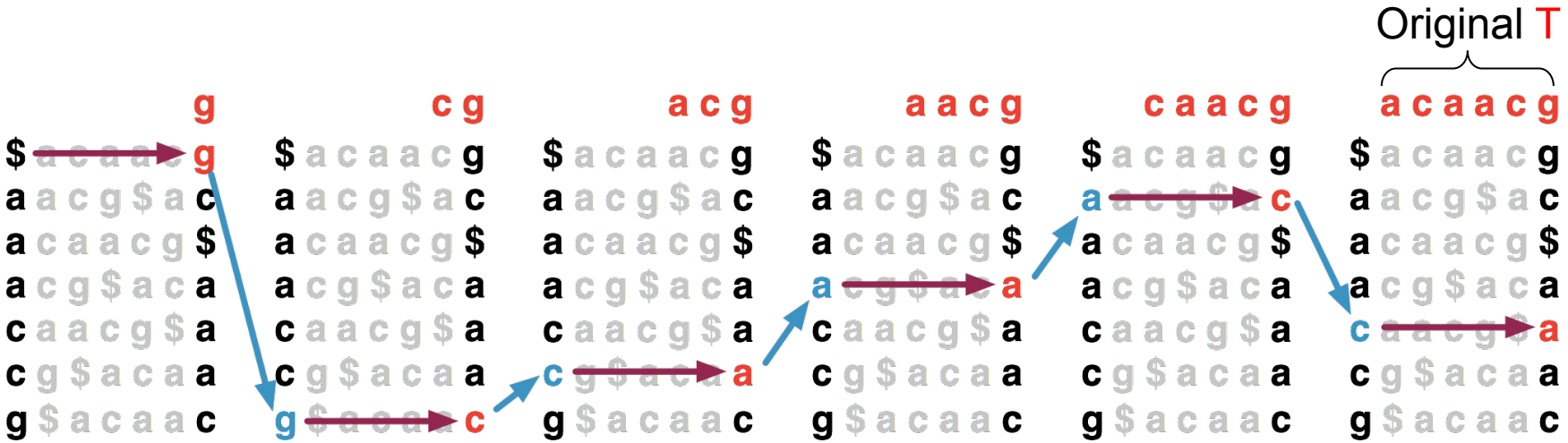
- BWT(T) is the index for T

LF Property implicitly encodes Suffix Array

A block sorting lossless data compression algorithm.
Burrows M, Wheeler DJ (1994) *Digital Equipment Corporation.* Technical Report 124

Burrows-Wheeler Transform

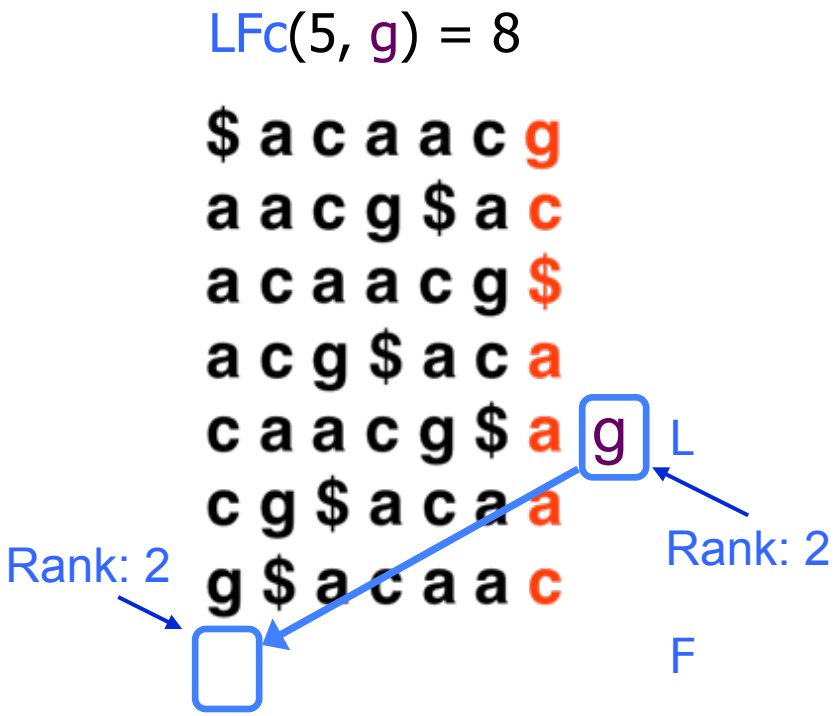
- Recreating T from BWT(T)
  - Start in the first row and apply **LF** repeatedly, accumulating predecessors along the way



[Decode this BWT string: ACTGA$TTA]

BWT Exact Matching

- **LFc**(r, c) does the same thing as **LF**(r) but it ignores r's actual final character and "pretends" it's c:

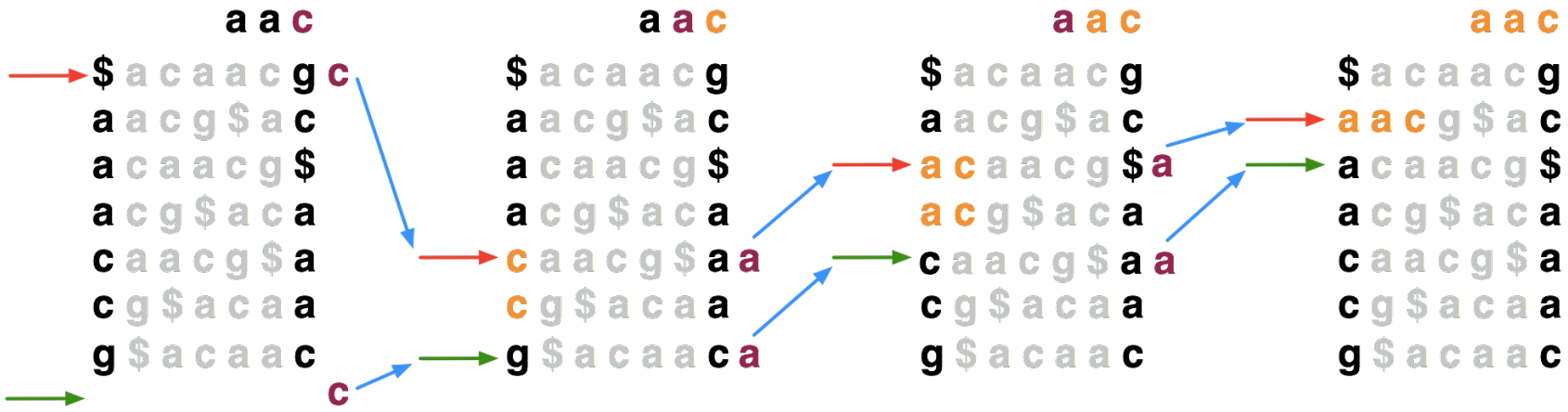


BWT Exact Matching

- Start with a range, (**top**, **bot**) encompassing all rows and repeatedly apply **LFc**:

  top = LFc(top, qc); bot = LFc(bot, qc)

  qc = the next character to the left in the query



Ferragina P, Manzini G: Opportunistic data structures with applications. *FOCS. IEEE Computer Society; 2000.*

[Search for TTA this BWT string: ACTGA$TTA]

Algorithm Overview

1. Split read into segments

Read

CCAGTAGCTCTCAGCCTTATTTTACCCAGGCCTGTA

Read (reverse complement)

TACAGGCCTGGGTAAAATAAGGCTGAGAGCTACTGG

Policy: extract 16 nt seed every 10 nt

Seeds

+, 0: CCAGTAGCTCTCAGCC

+, 10: TCAGCCTTATTTTACC

+, 20: TTTACCCAGGCCTGTA

-, 0: TACAGGCCTGGGTAAA

-, 10: GGTAAAATAAGGCTGA

-, 20: GGCTGAGAGCTACTGG

2. Lookup each segment and prioritize

Seeds

+, 0: CCAGTAGCTCTCAGCC

+, 10: TCAGCCTTATTTTACC

+, 20: TTTACCCAGGCCTGTA

-, 0: TACAGGCCTGGGTAAA

-, 10: GGTAAAATAAGGCTGA

-, 20: GGCTGAGAGCTACTGG

→ Ungapped alignment with FM Index

→ Seed alignments (as B ranges)

{ [211, 212], [212, 214] }

{ [653, 654], [651, 653] }

{ [684, 685] }

{ }

{ }

{ [624, 625] }

3. Evaluate end-to-end match

Extension candidates

SA:684, chr12:1955

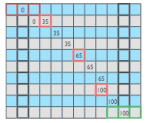
SA:624, chr2:462

SA:211: chr4:762

SA:213: chr12:1935

SA:652: chr12:1945

→ SIMD dynamic programming aligner



→ SAM alignments

```
r1   0    chr12    1936     0
     36M  *    0    0
     CCAGTAGCTCTCAGCCTTATTTTACCCAGGCCTGTA
     IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
     AS:i:0    XS:i:-2   XN:i:0
     XM:i:0    XO:i:0    XG:i:0
     NM:i:0    MD:Z:36   YT:Z:UU
     YM:i:0
...
```

Exact Matching Review

- E-value depends on length of genome and inversely on query length
  - $E = (n-m+1)/4^m$

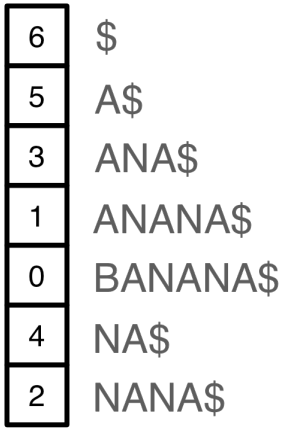
Brute Force (3 GB)

```
BANANA
BAN
  ANA
    NAN
      ANA
```

Naive

Slow & Easy

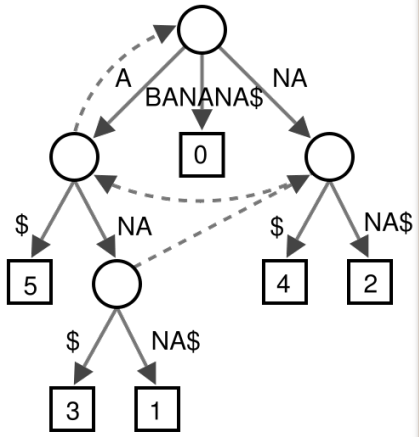
Suffix Array (>15 GB)



Vmatch, PacBio Aligner

Binary Search

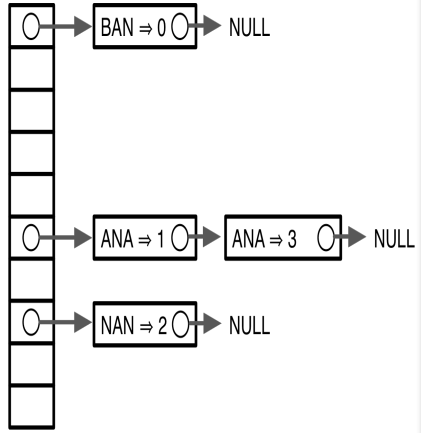
Suffix Tree (>51 GB)



MUMmer, MUMmerGPU

Tree Walking & DFS

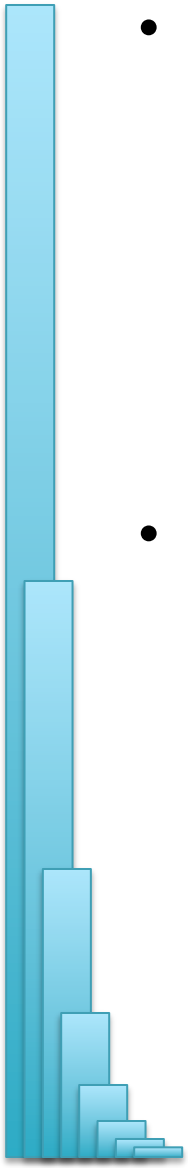
Hash Table (>15 GB)



BLAST, MAQ, ZOOM, RMAP, CloudBurst

Seed-and-extend

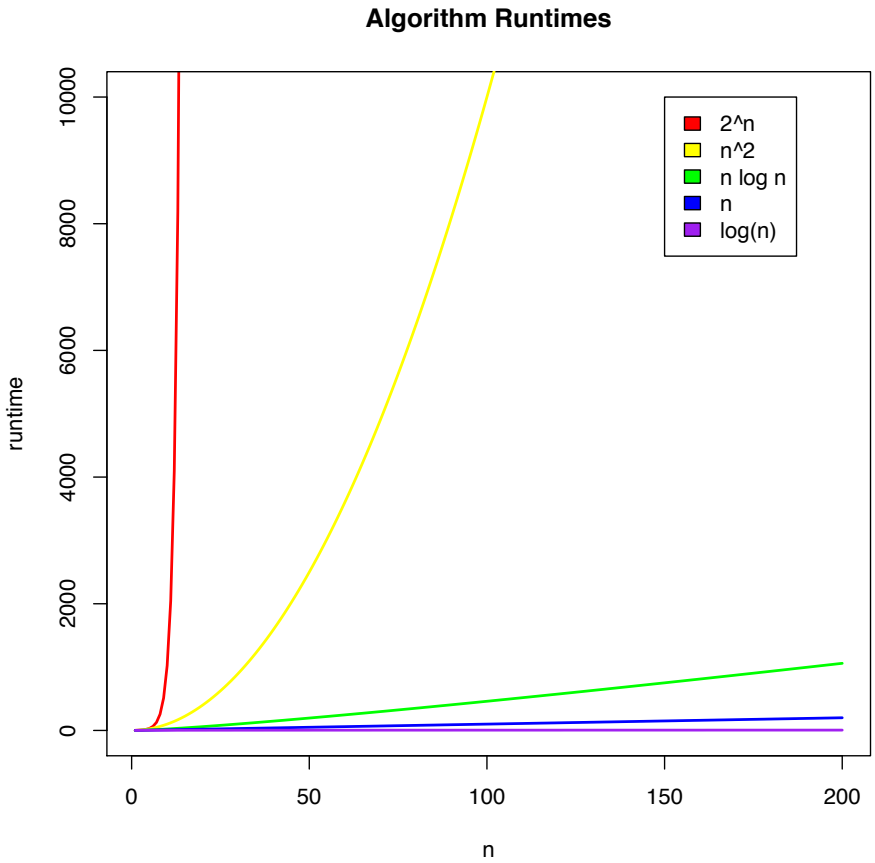
Algorithms Summary

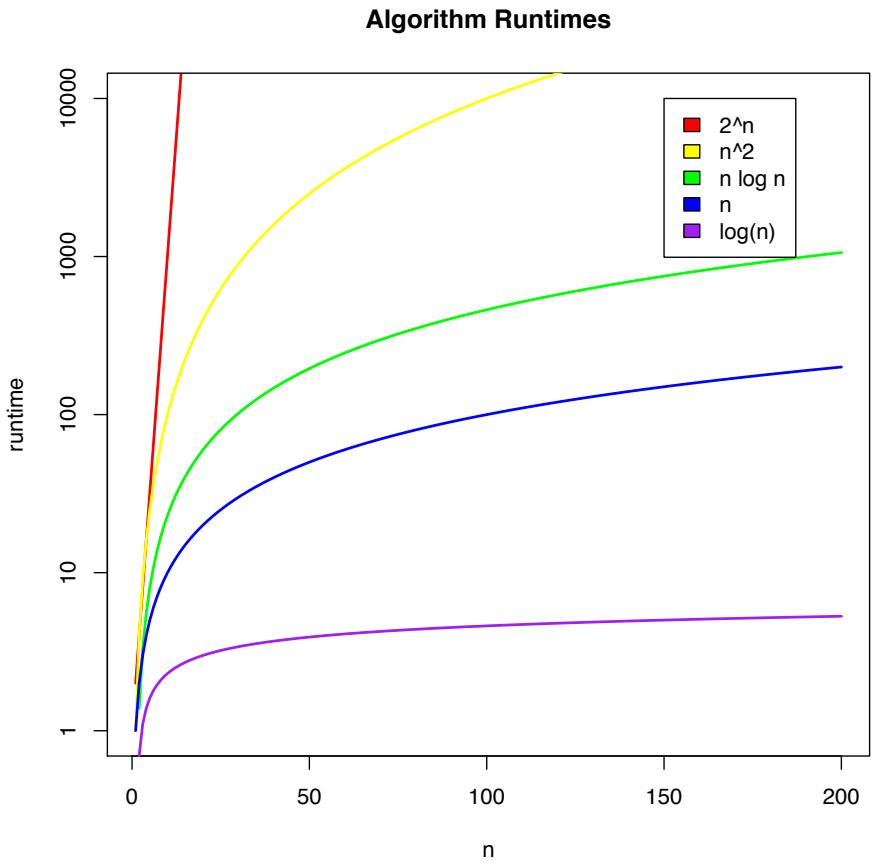


- Algorithms choreograph the dance of data inside the machine
  - Algorithms add provable precision to your method
  - A smarter algorithm can solve the same problem with much less work
  - Sequences are really fundamental to biology, learn the techniques to analyze them

- Techniques
  - Binary search: Fast lookup in any sorted list
  - Divide-and-conquer: Split a hard problem into an easier problem
  - Recursion: Solve a problem using a function of itself
  - Hashing: Storing sets across a huge range of values
  - Indexing: Focus on the search on the important parts
    - Different indexing schemes have different space/time features

Algorithmic Complexity





What is the runtime as a function of the input size?

Next Time

- In-exact alignment
  - Smith & Waterman (1981) *Identification of Common Molecular Subsequences*. J. of Molecular Biology. 147:195-197.
- Sequence Homology
  - Altschul SF, Gish W, Miller W, Myers EW, Lipman DJ (1990). *Basic local alignment search tool*. J of Molecular Biology. 215 (3): 403–410.
- Whole Genome Alignment
  - A.L. Delcher, S. Kasif, R.D. Fleischmann, J. Peterson, O. White, and S.L. Salzberg (1999) *Alignment of Whole Genomes*. Nucleic Acids Research (27):11 2369-2376.

Thank You!

http://schatzlab.cshl.edu
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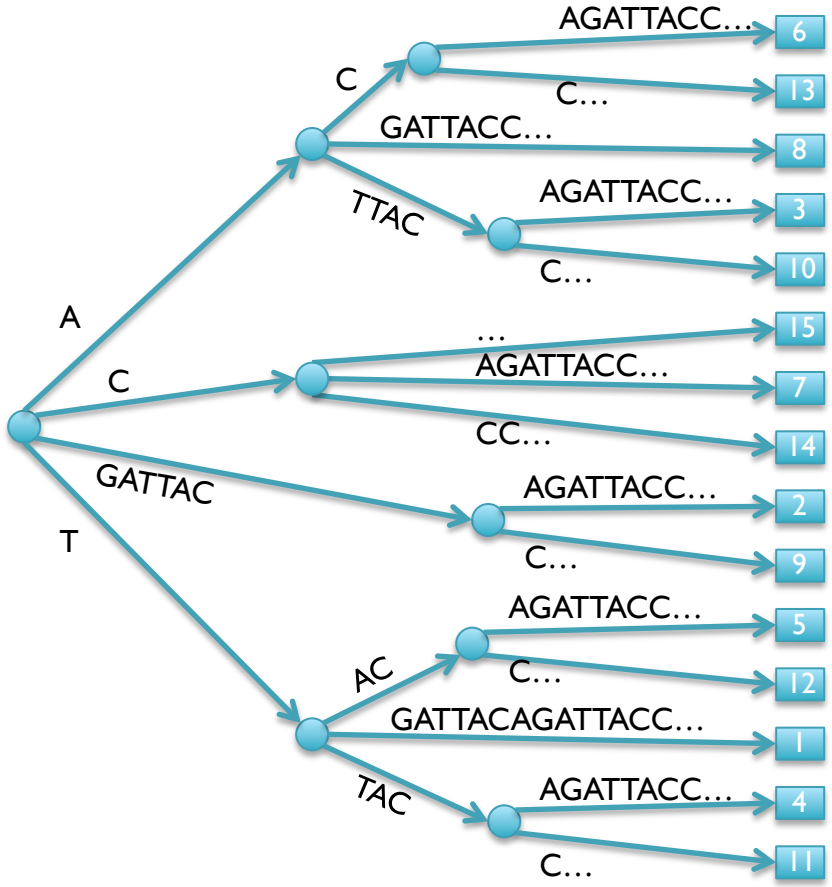
3. Suffix Trees (Optional)

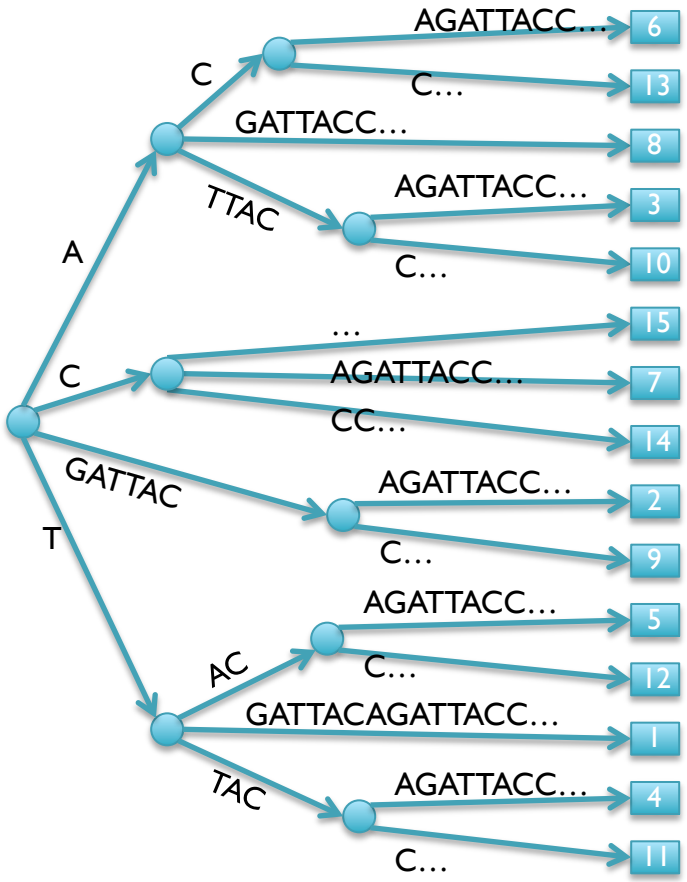
#	Sequence	Pos
1	ACAGATTACC…	6
2	ACC…	13
3	AGATTACC…	8
4	ATTACAGATTACC…	3
5	ATTACC…	10
6	C…	15
7	CAGATTACC…	7
8	CC…	14
9	GATTACAGATTACC…	2
10	GATTACC…	9
11	TACAGATTACC…	5
12	TACC…	12
13	TGATTACAGATTACC…	1
14	TTACAGATTACC…	4
15	TTACC…	11



- Suffix Tree = Tree of suffixes (indexes **all** substrings of a sequence)
  - 1 Leaf ($) for each suffix, path-label to leaf spells the suffix
  - Nodes have at least 2 and at most 5 children (A,C,G,T,$)

Suffix Trees Searching

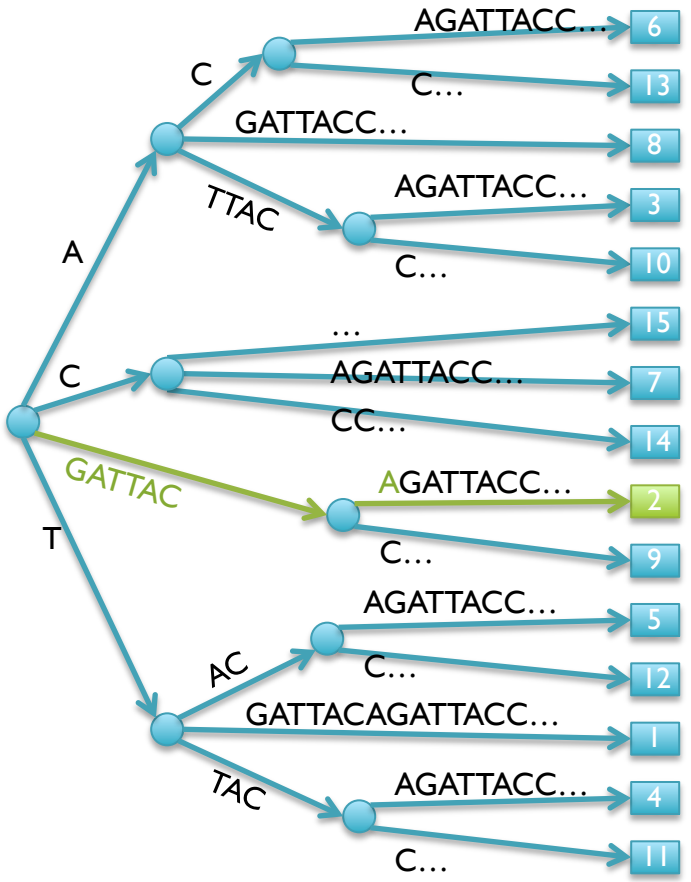
- Look up a query by "walking" along the edges of the tree
    - GATTACA



Suffix Trees Searching

- Look up a query by "walking" along the edges of the tree
  - GATTACA
  - Matches at position 2

```
WalkTree
    cur = ST.Root;
    qrypos = 0;
    while (cur)
        edge = cur.getEdge(q[qrypos]);
        dist = matchstrings(edge, qry, qrypos)
        if (qrypos+dist == length(qry))
            print "end-to-end match"
        else if  (dist == length(edge))
            cur=cur.getNode(edge[0]);
            qrypos+=dist
        else
            print "no match"
```



Suffix Trees Searching

- Look up a query by "walking" along the edges of the tree
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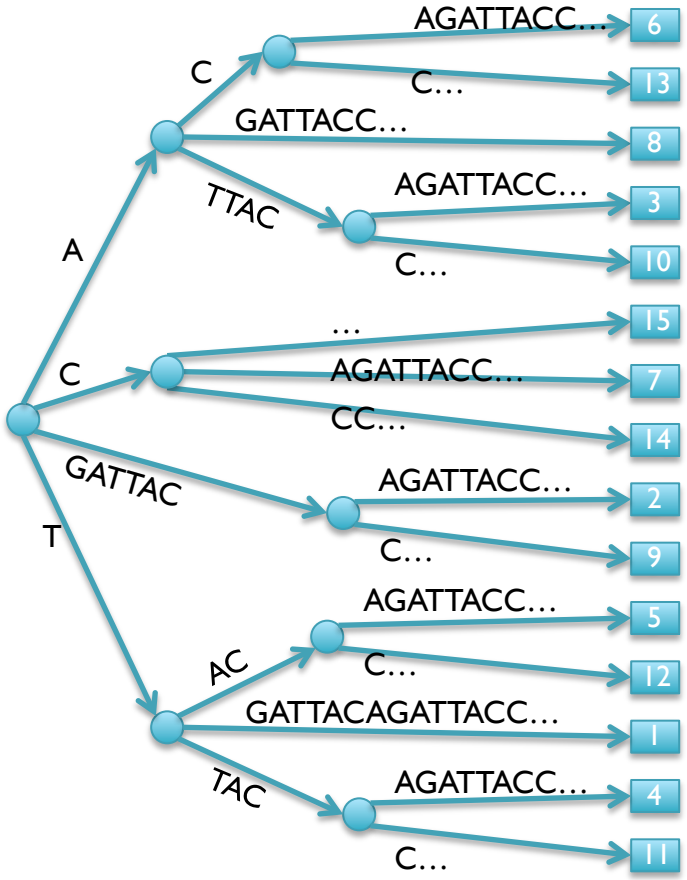
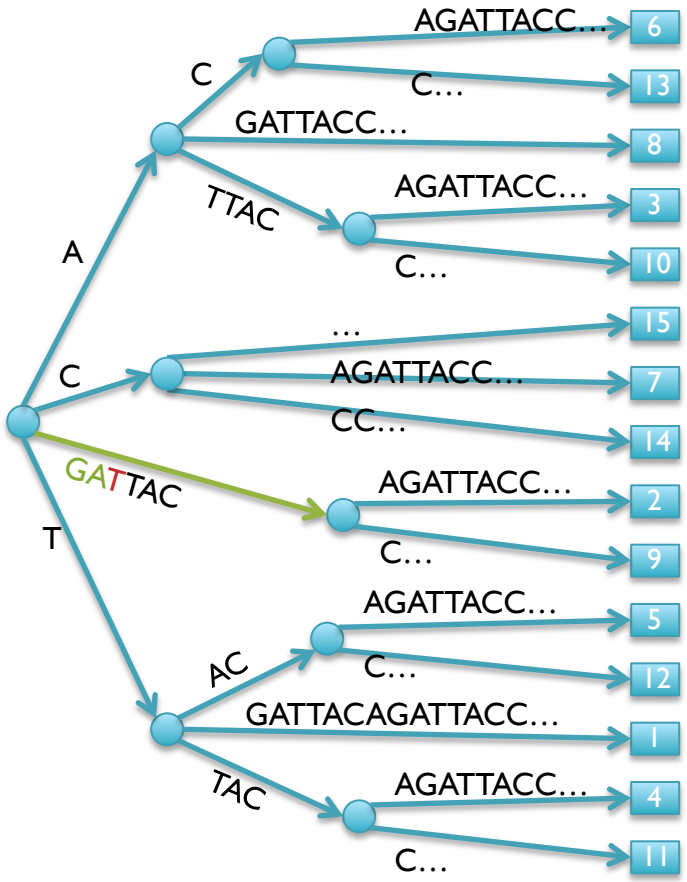


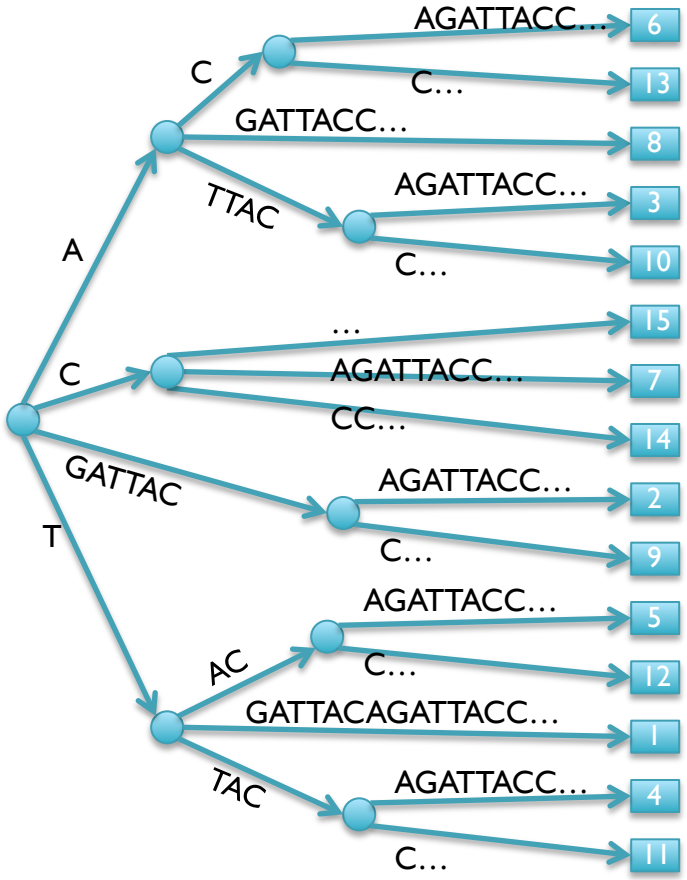
Suffix Trees Searching

- Look up a query by "walking" along the edges of the tree
  - GACTACA
  - Fell off tree – no match



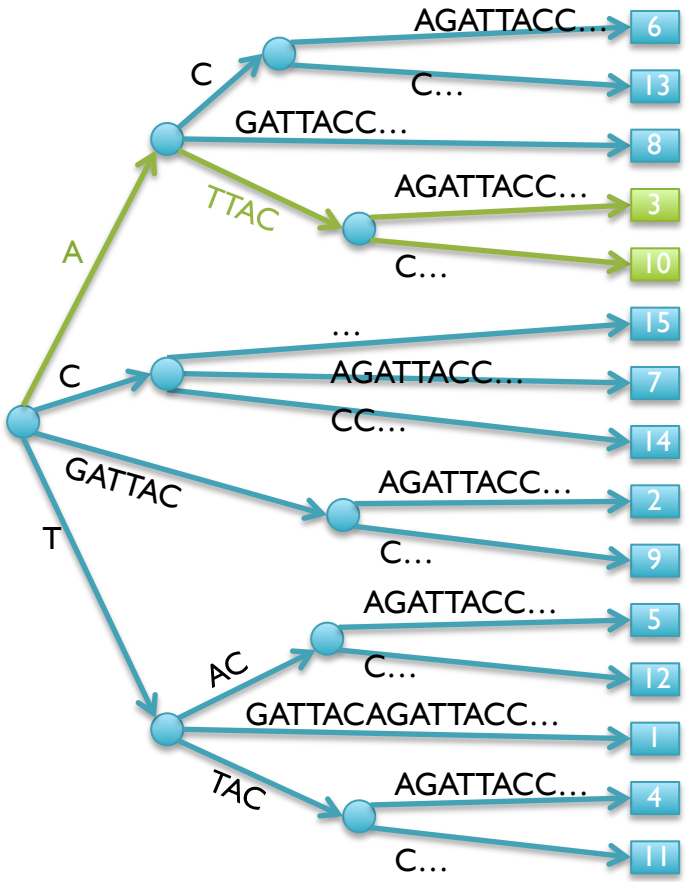
Suffix Trees Searching

- Look up a query by "walking" along the edges of the tree
  - ATTAC



Suffix Trees Searching

- Look up a query by "walking" along the edges of the tree
  - ATTAC
  - Matches at 3 and 10



  - Query Lookup in 2 phases:
    1. Walk along edges to find matches
    2. Walk subtree to find positions

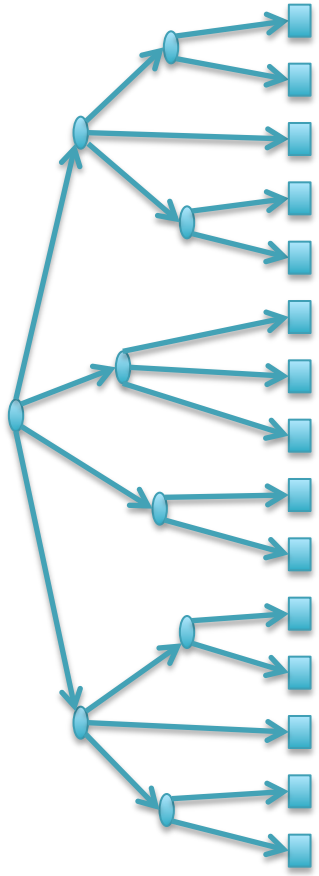
```
DepthFirstPrint(Node cur)
if cur.isLeaf
    print cur.pos
else
    foreach child in cur.children
     DepthFirstPrint(child)
```

[What is the running time of DFP
=> How many nodes does the tree have?]

Suffix Tree Properties & Applications

Properties

- Number of Nodes/Edges: O(n)
- Tree Size: O(n)
- Max Depth: O(n)
- Construction Time: O(n)
  - Tricky to implement, prove efficiency
  - Brute force algorithm requires $O(n^2)$



Applications

- Sorting all suffixes: O(n) [HOW?]
- Check for query: O(m)
- Find all z occurrences of a query O(m + z)
- Find maximal exact matches O(m)
- Longest common substring O(m)
- …

- Used for many string algorithms in linear time
  - Many can be implemented on suffix arrays using a little extra work